RIGOROUS ANALYSIS OF OPEN MICROSTRIP LINES OF ARBITRARY CROSS-SECTION IN BOUND AND LEAKY REGIMES†

Krzysztof A. Michalski and Dalian Zheng

Electromagnetics & Microwaves Laboratory
Department of Electrical Engineering
Texas A&M University, College Station, TX 77843

ABSTRACT

The problem of a microstrip line of arbitrary cross-section is solved by an integral equation technique in conjunction with the method of moments. The approach is general and can handle as special cases multiple strips and strips of finite or infinitesimal thickness. It applies to both the fundamental and higher-order modes, whether in the bound or leaky regime. Computed dispersion curves and modal current distributions are presented for several cases of interest and, where possible, are compared with published data.

1. INTRODUCTION

Although open microstrip lines have been analyzed by both spectral-domain [1,2] and space-domain [3,4] integral equation methods, these analyses are not easily extendable to lines whose upper conductor is not an infinitesimally thin, planar strip. The authors are only aware of two publications where more general cross-sections, namely rectangular [5] and circular [6], are considered. There is also a scarcity of results for higher-order modes in leaky regime [7].

In this paper, we present a rigorous dispersion analysis of open microstrip lines of arbitrary cross-sectional profile, based on the mixed-potential electric field integral equation (MPIE) [8,9]. We prefer the MPIE to several other possible forms of the electric field integral equation (EFIE), because it only requires *potential forms* of the Green's functions, which are less singular and converge faster than the *field forms* needed in other EFIEs. Another important advantage of the MPIE is its conformity with well-established numerical solution technqiues, originally developed for objects in free space [10].

2. FORMULATION

Consider a transmission line formed by an infinite, perfect conductor above a grounded dielectric slab, as illustrated in Fig. 1. The cross-sectional profile L of the conductor may be arbitrary, but—as indicated in Fig. 1—the solution procedure requires that it be approximated by straight line segments. Since the structure is of infinite extent and uniform along the y axis, we postulate that the associated fields, as well as the current density $\mathbf{J}$ on the upper conductor, vary with y as $\exp(-jk_y y)$, where $k_y = \beta - j\alpha$ is the propagation constant. In the absence of external excitation, the objective is to compute the propagation constants and the associated currents for the fundamental mode and for first few higher-order modes that can be supported by the structure of Fig. 1.

As was already mentioned, our approach is to formulate an MPIE and to solve it numerically by the method of moments [10]. We obtain the desired MPIE by specializing to the present two-dimensional case one of the general MPIEs developed recently by the authors [11]. The result is

$$\hat{\mathbf{n}} \times \{j\omega \mathbf{A}(l) + (\nabla_t - \hat{\mathbf{y}} jk_y)\,\Phi(l)\} = 0, \quad l \in L \tag{1}$$

where

$$\nabla_t = \hat{\mathbf{x}}\frac{\partial}{\partial x} + \hat{\mathbf{z}}\frac{\partial}{\partial z} \tag{2}$$

$$\mathbf{A}(l) = \int_L \underline{\underline{\mathbf{K}}}_A\,(l\,|\,l')\cdot \mathbf{J}(l')\,dl' \tag{3}$$

$$\Phi(l) = \int_L K_\phi\,(l\,|\,l')\;q(l')\,dl' \tag{4}$$

$$q(l) = \frac{j}{\omega}\,(\nabla_t - \hat{\mathbf{y}}\,jk_y)\cdot \mathbf{J}(l) \tag{5}$$

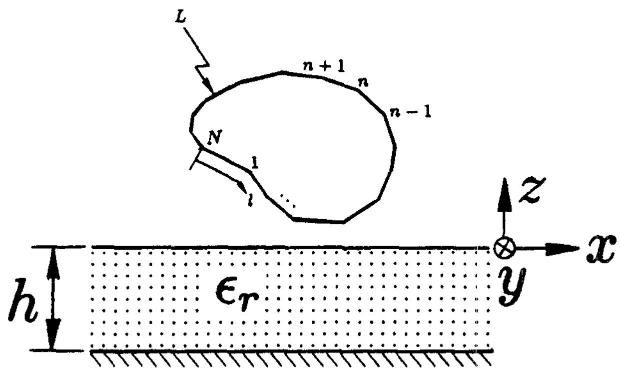

Fig. 1. Cross-sectional view of a cylinder of arbitrary shape above a grounded slab.

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The expressions for the dyadic kernel $\underline{\underline{\mathbf{K}}}_A$ and the scalar kernel K_ϕ comprise improper integrals, which are evaluated by numerical quadrature along suitably chosen paths in the complex k_x-plane, where k_x is the Fourier domain counterpart of x. These formulas are lengthy [12] and are ommited here due to lack of space.

The solution of (1) proceeds in a standard way [10]. We employ piecewise-constant (pulse) and piecewise-linear (triangle) basis functions to represent, respecitvely, the longintudinal and transverse components of $\mathbf{J}$. The same functions are used to "test" the equations in longitudinal and transverse directions. As a result, a homogeneous system of simultaneous algebraic equations is obtained for the current expansion coefficients. This system has nontrivial solutions for those values of k_y, which render its determinant vanish. Hence, to obtain the propagation constants of the various modes of the microstrip, a search is perfomed for the zeros of the determinant in the complex k_y-plane. For each propagation constant, the homegeneous system is solved for the corresponding current distribution.

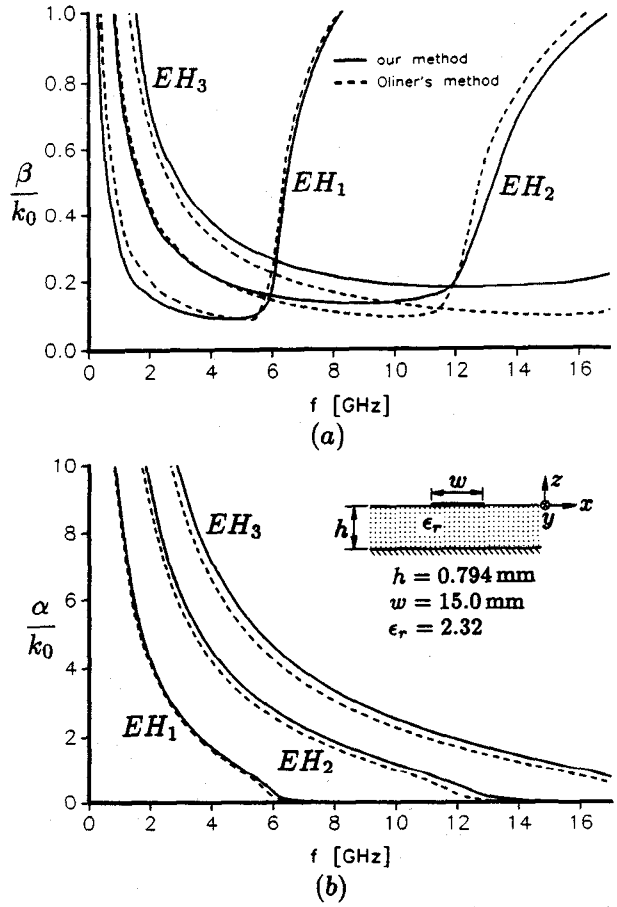

Fig. 2. Dispersion curves for the first three higher modes of an open, infinitesimally thin microstrip line. (a) Phase constants. (b) Attenuation (leakage) constants.

As was already alluded to above, the correct choice of the integration path with respect to the singularities in the spectral k_x-plane in evaluating the kernel elements is crucial for the success of this procedure [13]. It turns out that different paths must be selected for the bound regime (where the mode propagates unattenuated) and for the two leaky regimes (where the mode is attenuated due to loss of energy into the environment) [7,14]. It has been conjectured [15] that this is a possible reason why previous attempts to compute leaky microstrip modes by integral equation techniques have not been successful.

3. NUMERICAL RESULTS

In Fig. 2, we present sample dispersion characteristics of the first three higher modes in their leaky regime for a microstrip line previously analyzed by Oliner [7] using an elegant, but approximate, asymptotic approach [16]. The

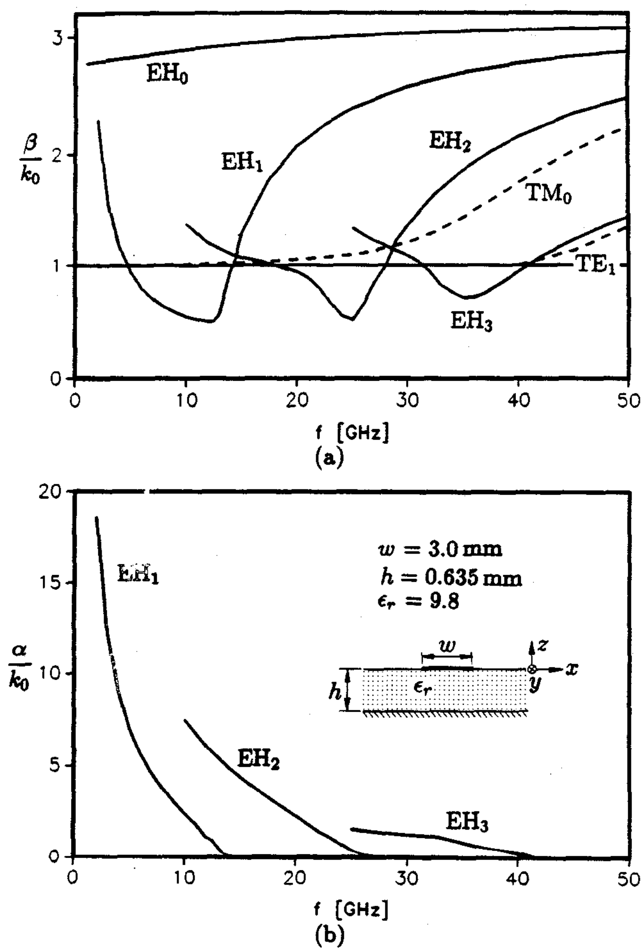

Fig. 3. Dispersion curves for the fundamental mode and the first three higher modes of an open, infinitesimally thin microstrip line. (a) Phase constants. (b) Attenuation (leakage) constants.

rigorous and asymptotic results are seen to agree quite well for the EH_1 mode; the agreement is somewhat less favorable for the higher-order modes. In Fig. 3, we present the dispersion curves for the fundamental and the first three higher modes in both bound and leaky regimes for a microstrip line with a higher dielectric constant.

In Fig. 5, we present the fundamental mode dispersion curves of microstrip lines of infinitesimal thickness and for transmission lines of rectangular and trapezoidal profiles (Fig. 4). For the latter, we show sample current distributions at $f = 6\,\text{GHz}$ in Fig. 6.

In Fig. 7, we present the fundamental mode effective dielectric constant, defined as $\epsilon_{eff} = (\beta/k_0)^2$, for a circular-wire transmission line. In this figure, Faché and De Zutter's results [6] are also plotted for comparison.

4. CONCLUSIONS

For the first time, a rigorous solution has been presented of a microstrip transmission line of arbitrary cross-section. Dispersion curves and current distributions have been computed for transmission lines of various shapes and, where possible, compared with available data.

REFERENCES

[1] T. Itoh and R. Mittra, "Spectral domain approach for calculating the dispersion characteristics of microstrip lines," *IEEE Trans. Microwave Theory Techn.*, vol. MTT-21, pp. 496–499, July 1973.

[2] M. Kobayashi and F. Ando, "Dispersion characteristics of open microstrip lines," *IEEE Trans. Microwave Theory Techn.*, vol. MTT-35, pp. 101–105, Feb. 1987.

[3] J. M. Grimm and J. S. Bagby, "Alternative integral formulation for analysis of microstrip transmission lines," *Digest of the URSI Nat'l Radio Sci. Meeting*, p. 113, Boulder, CO, Jan. 1988.

[4] N. Faché and D. De Zutter, "Rigorous full-wave space-domain solution for dispersive microstrip lines," *IEEE Trans. Microwave Theory Techn.*, vol. MTT-36, pp. 731–737, Apr. 1988.

[5] S. Tedjini, N. Daoud, D. Rauly, and E. Pic, "Analysis of MMICs with finite strip thickness and conductivity," *Electron. Lett.*, vol. 24, pp. 965–966, July 1988.

[6] N. Faché and D. De Zutter, "Full-wave analysis of a perfectly conducting wire transmission line in a double-layered conductor-backed medium," *IEEE Trans. Microwave Theory Techn.*, vol. MTT-37, pp. 512–518, Mar. 1989.

[7] A. A. Oliner, "Leakage from higher modes on microstrip line with application to antennas," *Radio Sci.*, vol. 22, pp. 907–912, Nov. 1987.

[8] K. A. Michalski, "The mixed-potential electric field integral equation for objects in layered media," *Arch. Elek. Übertragung.*, vol. 39, pp. 317–322, Sept.-Oct. 1985.

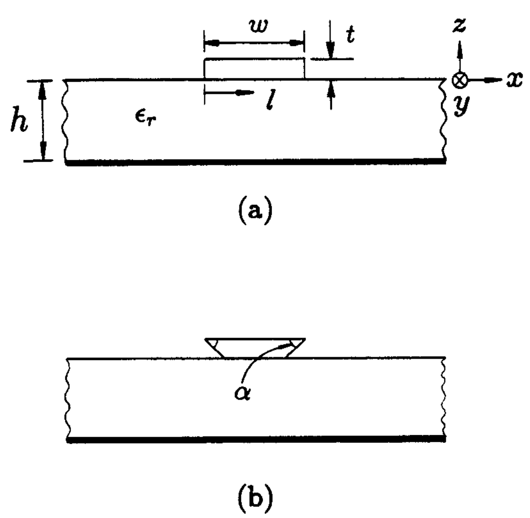

Fig. 4. Cross-sectional view of a microstrip line of finite thickness. (a) Rectangular strip. (b) Trapezoidal strip.

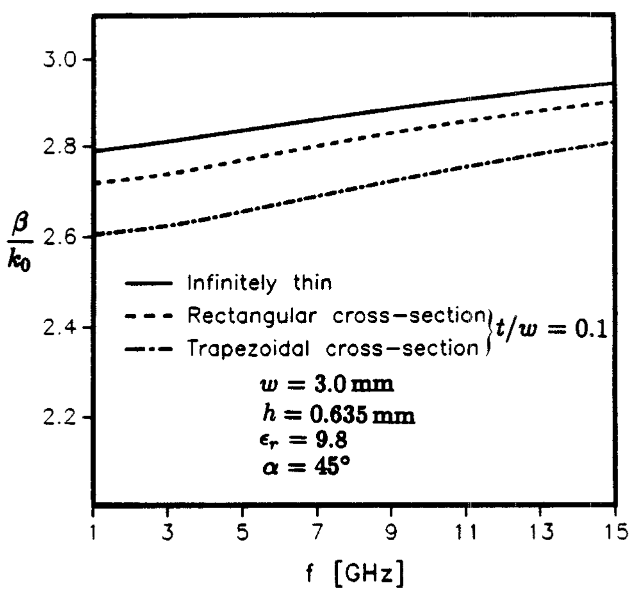

Fig. 5. Fundamental mode dispersion cuves for microstrip lines of infinitesimal thickness and for microstrip lines of rectangular and trapezoidal cross-sections.

[9] J. R. Mosig, "Arbitrarily shaped microstrip structures and their analysis with a mixed potential integral equation," *IEEE Trans. Microwave Theory Techn.*, vol. MTT-36, pp. 314–323, Feb. 1988.

[10] A. W. Glisson and D. R. Wilton, "Simple and efficient numerical methods for problems of electromagnetic radiation and scattering from surfaces," *IEEE Trans. Antennas Propagat.*, vol. AP-28, pp. 593–603, Sept. 1980.

[11] K. A. Michalski and D. Zheng, "Electromagnetic scattering and radiation by surfaces of arbitrary shape in layered media, Part I: Theory," *IEEE Trans. Antennas Propagat.*, submitted.

[12] D. Zheng, "Radiation, scattering and guidance of electromagnetic fields by conducting objects of arbitrary shape in layered media," Ph.D. dissertation, Dept. Elec. Eng., Univ. Mississippi, Dec. 1988.

[13] K. A. Michalski and D. Zheng, "On the choice of branch cuts and integration paths in the analysis of open microstrip structures," *Electron. Lett.*, submitted.

[14] H. Ermert, "Guiding and radiation characteristics of planar waveguides," *IEE Microwave, Optics and Acoustics*, vol. 3, pp. 59–62, Mar. 1979.

[15] K. A. Michalski and D. Zheng, "On the leaky modes of open microstrip lines," *Microwave and Optical Technology Letters*, vol. 2, pp. 6–8, Jan. 1989.

[16] E. F. Kuester, R. G. Johnk, and D. C. Chang, "The thin-substrate approximation for reflection from the end of a slab-loaded parallel-plate waveguide with application to microstrip patch antennas," *IEEE Trans. Antennas Propagat.*, vol. AP-30, pp. 910–917, Sept. 1982.

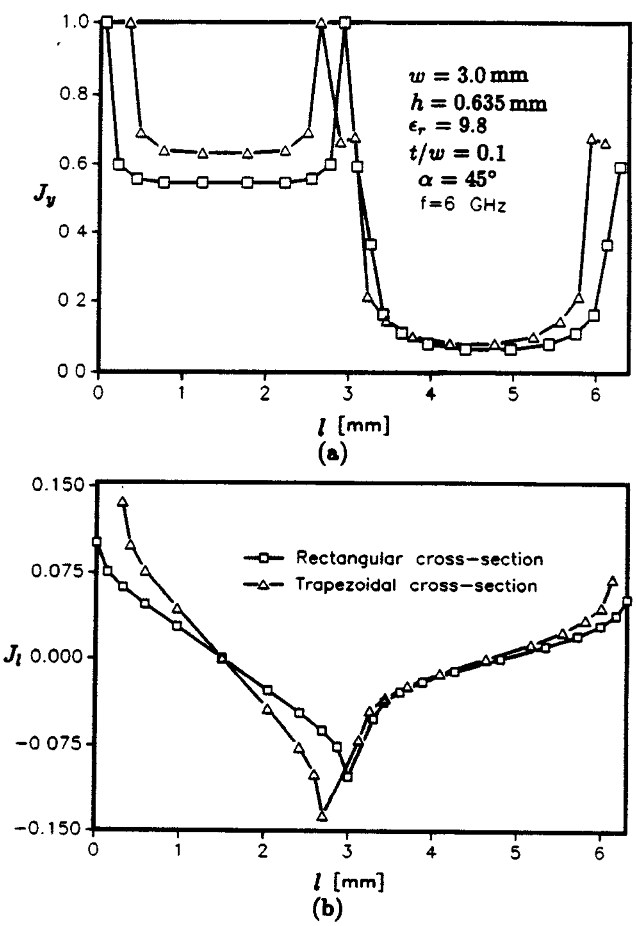

Fig. 6. (a) Longitudinal and (b) transverse current distributions of the fundamental mode for transmission lines of rectangular and trapezoidal cross-sections.

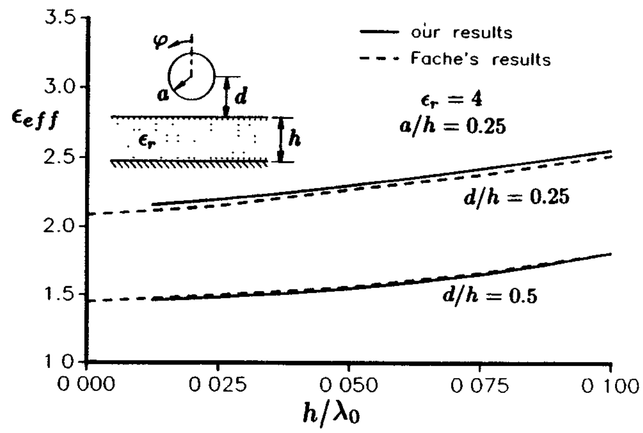

Fig. 7. Fundamental mode effective dielectric constants for a circular-wire transmission line.